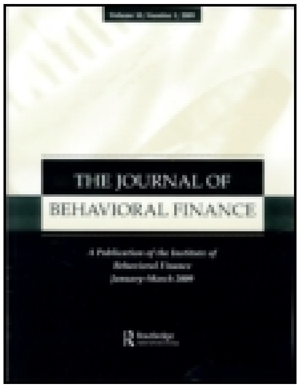

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Explaining What Leads Up to Stock Market Crashes: A Phase Transition Model and Scalability Dynamics

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Mathematical descriptions of financial markets with respect to the efficient market hypothesis (EMH) and fractal finance are now equally robust but EMH still dominates. EMH and other current paradigms are extended to accommodate situations having higher information complexity and interactions coupled with positive feedback. The "herding behavior" literature in finance marks a significant recognition that interdependent trader behavior may result in deviation from normal distribution of returns, as does "chartist" trading. Further legitimization of the separate-but-equal status of EMH and fractal finance is pursued. Research on the nonlinear models giving theoretical underpinning to equations representing mirror markets as complex dynamical systems is encouraged. Why some herding- and chartist-behaviors scale up and then die off whereas others result in significant crashes is explained. The buildup to the 2007 liquidity crisis offers an example of nonlinear scale-free dynamics. Concepts from complexity science, econophysics, and scale-free theory are used to offer further explanation to physicists' mathematical treatments.

Keywords: Market crash, Phase transition, Imitation, Herding, Self-organization

INTRODUCTION

Although Gene Fama championed the publication of Benoit Mandelbrot's article in the *Journal of Business* in 1963, he also wrote a critique and stayed on the path of the three dominant finance paradigms: the efficient market hypothesis (EMH) (e.g., Fama [1970]), the capital-asset pricing model (CAPM) (Sharpe [1964], Lintner [1965], Black [1972]), and the Black-Scholes [1973] options-pricing model. In parallel, Mandelbrot [1970, 1982, 1997, 1999, 2001] and Mandelbrot and Hudson [2004] kept applying his fractal geometry ideas to stock markets. His views have more recently been picked up by others (e.g., Peters [1991, 1994], Rosser [2000], Sornette [2003a], Malevergne and Sornette [2005], Jondeau et al. [2007], Calvet and Fisher [2008]). Over the years these two competing schools of thought have remained contentious, arguing about the same turf, with little basis of reconciliation apparent. For example, in his 2004 book with Hudson, Mandelbrot says: "Modern financial theory is founded on a few, shaky myths that lead us to underestimate the real risk of financial markets." He adds: "Orthodox financial theory is riddled with false assumptions and wrong results" (pp. ix, x). Cooper [2008, p. 11) says: "Despite overwhelming evidence to the contrary, the Efficient Market Hypothesis remains the bedrock of how conventional wisdom views the financial system. . ." According to Fama [1998], however, until a new and better paradigm is put forth, one cannot criticize EMH/CAPM. Fama reduces behavioral finance—and trading dynamics—to anomalies and over-/underreaction episodes that are normally distributed.

The mathematical descriptions of financial market behavior by each school are now equally robust. Still, the EMH, CAPM, and options pricing paradigms have successfully remained at the center of market analysis for most researchers in the finance community, even though the fractal school has continued to grow in numbers of participants and depth of mathematical analysis (e.g., Adler, Feldman and Taqqu [1998], Rachev and Mittnik [2000], Mantegna and Stanley [2000], Malevergne and Sornette [2005]). Since we have just passed through another of what Alan Greenspan [2008]

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recently termed a "once-in-a-century" market crash, our concern about what sets off unusual volatility sequences and occasional extreme crashes is surely timely and calls for further analysis of when and why EMH market trading shifts into behaviors better fitting fractal mathematics.

Following EMH, we first distinguish between the *rational* and *noise* traders who, acting *independently* of each other, create efficient markets and Gaussian distributions of returns. First, we note that there are three kinds of equally relevant trading behaviors that our models and theories have to account for: (a) There are two kinds of trader behavior in which they act *independently* of each other, which are excellently represented by the formalizations of EMH—which includes both *rational* and *noise* traders; and (b) A third kind of trader behavior also exists—where trader behaviors are in fact *interdependent*. We will call this the *high-risk* trading category; it occurs when traders resort to *rule-driven behavior*—for example, *chartists*, *herding*, *information cascades*, etc.—in short, *interdependent traders* (note that this behavior can be modeled as the rate of imitation in the logistic equation). Second, we note, however, that Sornette and his colleagues are content to draw explanatory closure when they fit an equation to a few parameters representing log-periodic oscillations of prices (Sornette [2004]). A number of quantitative researchers now fit extreme events into power-law distributions of price changes to formalize nonlinear patterns during periods of extreme volatility (e.g., Jondeau et al. [2007], Calvet and Fisher [2008]). But our question remains: *What are the various kinds of interdependent trader-induced causes that scale up into the fractal volatility sequences and occasional extreme events (crashes) we see in stock market behaviors?*

We do not wish to replace the EMH/CAPM school but rather extend the existing framework to accommodate situations with higher information complexity, interactions with positive feedback (Minsky [1982, 1986]), and extreme events that cannot be simply explained by presuming EMH's independent-additive data points, and normal distributions. For example, the development of the "*herding behavior*" literature in finance (e.g., Banerjee [1992], Bikhchandani et al. [1992], Prechter [1999], Brunnermeier [2001], Rook [2006]) marks a significant recognition that interdependent trader behavior may result in skew distributions of stock market prices and, therefore, offers the first underlying explanation of behavior that may begin as Holland's [1995, 2002] "*tiny initiating events*" but later scales up into extremes. We also follow Wolfson [2002], Cooper [2008], Phillips [2008], and Foster and Magdoff [2009] in drawing on Hyman Minsky's works of 1982 and 1986 and 2008. Building on this, we introduce two so-called "scale-free theories" to explain why some tiny initiating events scale up into extremes while many others do not. In all of these, interdependent trader behavior is the critical element. Our scale-free theories come from a range of disciplines (Andriani and McKelvey [2009]).

We attempt to further legitimate the separate-but-equal status of EMH and fractal finance (FF) by going beyond classic financial asset pricing theory so as to account for findings of behavioral aberrations arising where unrealistic assumptions are made of unbounded rationality and independent judgments among investors about future payoffs and choices made solely on those anticipated payoffs (Fama and French [2007]). We begin with an analogy among the three states of a physical system (solid, liquid, and gas) and stock market dynamics at a "microscopic" level, where an individual trader has only three possible actions: selling, buying or waiting; we call this the *Triple Point*. Market crashes—our *Critical Point*—have now been explained theoretically and measured quantitatively in a variety of ways. In between the two Points, bubble-buildups begin if and when traders gravitate toward herding behaviors based on interaction and learning about the value of what becomes the dominant buy-sell trading rule. Sornette et al. [1996] reveal log-periodic oscillations of index prices before significant drawdowns and suggest a phase transition model to explain trading behavior between the two Points. According to this model, the phase transition occurs at a *tipping point*—***R*1** in Figure 3—at which traders begin herding, which eventually leads to a crash.

We use the "volatility autocorrelation function," Hurst exponent, H, and power laws (PLs) to identify the tipping point between the Triple and Critical Points. In doing this, we build from Sornette et al.'s empirical work (Yan, Woodard and Sornette [2010]) underlying his and our use of log periodicity and $H > \frac{1}{2}$ to indicate the positioning of our tipping-point indicator. This allows us to identify the tipping point between the *Triple Point* (where greed, risk, and noise are balanced à la Bachelier's random walk of stock markets (PhD dissertation in 1900) and Fama's EMH [1970]) and the beginning of herding-style trading—which dates back to Henri Poincaré [1914]—that instigates bubble-buildups toward a market crash at the *Critical Point*.

The objective of this paper is to encourage research on the nonlinear models by giving some theoretical underpinning to the equations that mirror markets as complex dynamical systems. Instead of just an "either-or" view of EMH or FF, we especially focus on dynamics causing traders to shift from one regime to the other. Research about extreme events and underlying scale-free dynamics is of particular interest for the overall understanding of markets functioning as complex systems reveal their characteristics under stress better than in normal conditions. Baum and McKelvey [2006] also reveal the potential advantage of extreme value theory in modeling management phenomena and the increasing popularity in financial applications. Andriani and McKelvey [2007] reveal how misleading are the assumptions behind the econometric methods involving linear multiple regression.

We use a physical analogy to help define the axes of our "Financial Markets Phase Diagram" (Figure 3). In this diagram we depict and define the *Triple Point* where we see EMH-driven behaviors and the *Critical Point* at which major market crashes occur, that is, 1929, 1987, and 2007+. We then turn our attention toward explaining the various kinds of nonlinearities in trader behavior occurring as "market mania" develops. We define four kinds of nonlinearities. Next, we use autocorrelation functions, log-periodicity, and the Hurst exponent to identify the ***R*1** tipping point. Finally, we apply concepts from econophysics and scale-free theory to zero in on the underlying causes of these market nonlinearities. A conclusion follows.

THE PHYSICAL BASIS OF PHASE TRANSITION (AND CRITICALITY?)

In the physical sciences, a phase space depicts the set of states of a macroscopic physical system that has relatively uniform chemical composition and physical properties. For example, for water the three phases (solid, liquid, and gas) are defined by temperature/pressure combinations. The different phases of a system may be represented using a phase diagram (Figure 1). The *Triple Point* is the combination of temperature and pressure that permits the almost simultaneous co-existence of the three phases in dynamic equilibrium.[1] A phase transition, (or phase change) is the transformation of a thermodynamic system from one phase to another. At the *Critical Point*, a second-order phase transition occurs leading to the disappearance of the phase boundary and the presence of a super-critical liquid-or-gas state.

Thus, when a system transitions from one phase to another, there will generally be a stage where the free energy is nonanalytic. The free energies on either side of the transition are two different functions, so one or more thermodynamic properties will behave differently after the transition. The property most commonly examined in this context is the heat capacity of the substance. During a transition, heat capacity may become infinite, jump abruptly to a different value, or exhibit a "kink" or discontinuity in its derivative, that is, experience an abrupt sudden change in heat capacity with only a small change in temperature (Figure 2). An example of phase transition at constant pressure and increasing temperature causes heat capacity solids to increase; a phase transition to gas causes the heat capacity to decrease. Heat capacity and the compressibility define analogous concepts with *respect to T and P*; therefore phase transition from gas to liquid at constant temperature will be accompanied by change in compressibility.

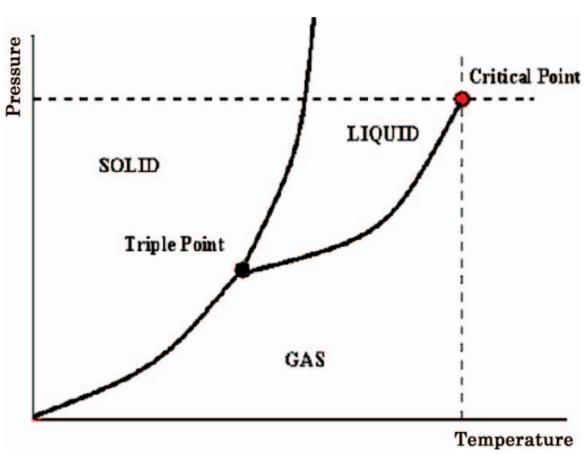

FIGURE 1 Phase diagram. (Color figure available online).

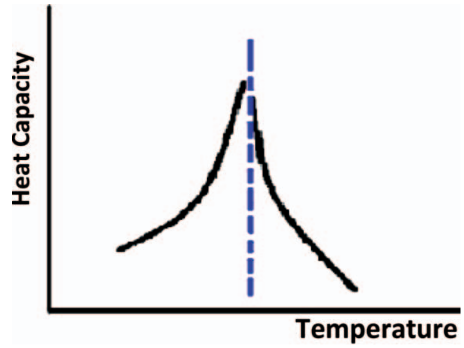

FIGURE 2 Heat capacity discontinuity. (Color figure available online).

FROM PHYSICS TO FINANCE: A PHASE-TRANSITION MODEL

In an auction, market price is determined by demand and supply, that is, buy and sell orders. The balance between the two we define as net demand. Balanced net demand or "0" will determine a "wait" phase in the market. Transactions do not impact price; small fluctuations in the price are similar to low vibration of molecules in the solid state. When the demand is positive, the market is in "buy phase" and the parallel with gas may be viewed as pressure on the price from below. The region of negative net demand, a "sell" market, is characterized by pressure on the price from above (liquid phase). Autonomous agents with heterogeneous information and bounded analytical abilities place their orders to buy and sell securities. At a macro level, the balance of these trading orders determines the phase of the market; that is, the buy/sell decision ratio divides the plane. At the *Critical Point* the market crashes as the trade orders are predominantly on the sell side, as Sornette [2003a] points out. The market "*phase diagram*" should be able to explain the formation of the net demand regions with appropriately chosen variables on the X and Y axes of Figure 3.

Defining the Axes

The Y-axis measures the risk of a security with respect to its value based on fundamentals. Closer to the origin—the region of certainty—the risk level in the fundamentals (indicated by volatility) is larger, which leads to underpricing of stocks. In equilibrium at **TPy1**, risk is properly incorporated in analyses when the ratio of risk value

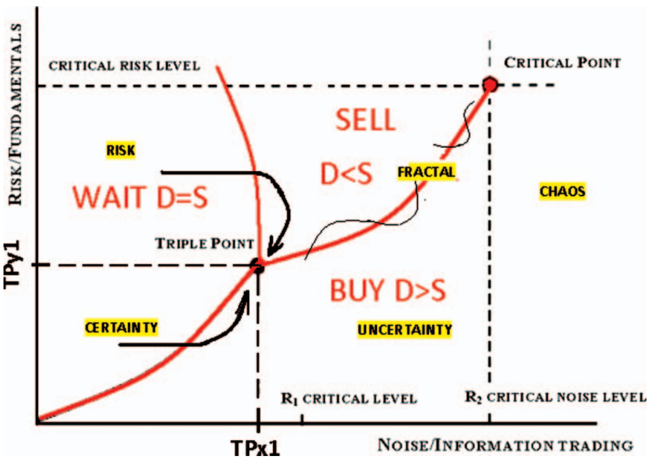

FIGURE 3 Financial markets phase diagram. (Color figure available online).

÷ fundamental value = 1; hence, the price is "fair" and governed by EMH. However, going higher up the Y axis increases risk-taking behavior, which underestimates volatility of the firm's underlying fundamentals, leading to overpricing.

Returning to our two types of investors (rational and noise), rational investors evaluate all available information and make their trading decisions based on the ratio of risk and return, defining underpriced (buy) and overpriced (sell) stocks. The Y-axis of our financial markets phase diagram measures these decisions by a risk/fundamentals ratio, which = 1 at **TPy1**. In the base condition (when system complexity is minimal and rational traders hold uniform assumptions), the opposite side of a trade is attributed to liquidity traders in the EM paradigm; sell pressure on the price increases with increase of risk (on the Y-axis) as investors are predominantly risk averse.

On the X-axis, we replace temperature with the noise-to-information trading ratio. In the physical phase transition diagram, temperature was defined in terms of the Second Law of Thermodynamics, which deals with entropy. Since entropy is a measure of the disorder in a system, we believe that disorder in the market can be measured by the ratio of noise-to-information trading. Information trading implies investors can properly process information and act rationally. As we already mentioned, if *all* investors are rational and all trading decisions are information based, we will have homogeneous agents, one-sided trade orders, and a "halt" in the market. Conversely, as noise trading becomes increasingly evident in the market, rationality recedes and disorder increases.

The dimensions that we have chosen for our axes have been used in some nonlinear financial models; for example, the asset price diffusion process is explained by the ratio of noise-to-rational traders and the distance between fundamental-to-actual price in Chiarella [1992], Day and Huang [1990], and Lux [1995]. These two dimensions will also allow us to place into the asset pricing framework the log-periodic oscillations of index prices before crashes, as reported by econophysicists (e.g., Johansen et al. [2000], Sornette [2003a, b]).

The three "market phases" (wait, buy, and sell) are determined by the demand/supply ratio of trading orders. Decisions behind these orders should be examined in light of the information complexity of the marketplace (certainty toward the origin; increasing noise-induced uncertainty to the right; and increasing risk going upward). Our model suggests that, given randomly arriving information, investors apply a probabilistic approach to decision making. In this market environment, prevailing rationality and information transparency allow quick adjustment of prices. Transparent, freely available information is accurately incorporated in stock prices in line with the Efficient Market Hypothesis while balance between information and liquidity traders is required to maintain dynamic equilibrium around the Triple Point. We define the region toward the origin, which has minimal noise and low risk, as the region of "certainty": securities with low risk have prices based on fundamentals (Y-axis) and good information (i.e., low noise) (X-axis).

In normal markets with heterogeneous independent traders and random noise, prices adjust to fundamental (certain) value; anomalies are short lived. Under some conditions (e.g., new technology, trading rules, or formulas such as derivatives), noise in the market increases and creates information ambiguity. Therefore uncertainty-averse traders switch to nonprobabilistic approaches to decision making.

Moving up the y-axis from the region of certainty, traders enter the EM region at **TPy1** (risk value and fundamentals value are balanced, i.e., risk/fundamentals = 1); higher idiosyncratic "risk" is correctly incorporated and discounted by rational investors. Prices experience downward pressure, thereby increasing expected returns for the higher levels of risk. It is here that we see the development of faith in new technologies and high-risk trading and portfolio management strategies, increased use of derivatives, and so forth. "The rational" and "educated" use of these does not lead to persisting anomalies in the underlying asset prices that could lead to herding and bubble building. The market sustains equilibrium around the Triple Point; efficiency prevails as long as rational decision making based on probabilistic approach is feasible.

Moving horizontally from the region of certainty to the right, traders enter the EM region on the x-axis at **TPx1**; they enter the region of higher information complexity and uncertainty "distinct from the familiar notion of risk," as defined by Knight [1921]. In uncertain situations, many decision makers still try to find and then bet on unambiguous events rather than settling for ambiguous ones (Basili and Zappia [2003]). This contradicts the appropriateness of probabilistic decision making. Shackle [1949] developed a theory opposing the subjective probability approach. Among the different theories of decision making under uncertainty, the info-gap theory has been applied in economics. It is a nonprobabilistic decision theory seeking to optimize robustness to failure or opportunities for windfall profits. Zhang [2006] investigates the role of information uncertainty in price-continuation anomalies and cross-sectional variations in stock returns; he shows that short-term price continuation is due to investor behavioral biases, which result in greater price drift when there is greater information uncertainty.

We argue that given the random nature of good/bad news, information uncertainty alone cannot produce a bubble buildup as greater information uncertainty should randomly produce relatively higher expected returns following good news and relatively lower expected returns following bad news. This region of uncertainty around the Triple Point is not the focus of our paper. It is not on the bubble buildup path characterized by log-periodic oscillation, which leads to crashes. We will concentrate on the part of the model that reveals the dynamics of precrash market, that is, the "*fractal*" region of Figure 3.

In the region of high levels of noise and high risk, that is, approaching the Critical Point, imitative behavior leads to power laws and fractals in the security pricing, self-organization, and emergence of structure instigates departure from efficiency. This region is also characterized by the high speed of trading and frequent reversals in the price function. Similarly, Brock and Hommes [1998] claim that large segments of noise and risk traders are likely to destabilize prices.

Triple-Point Dynamics

If trading is at both **TPy1** and **TPx1**, overall market state is defined as "wait" when net demand is balanced; that is, shares may exchange hands but this will not affect the price. In this "phase" we have two regions that are unstable (i.e., oscillating fairly rapidly), which quickly settles at the Triple Point. This attractor basin is characterized by correctly priced securities (Y-axis: risk/fundamentals = 1) and balance between noise and information trading (X-axis: noise/information = 1).

The lower left corner has thin trading as rational investors are willing to buy the underpriced securities, but there are very few noise traders who will sell at this price. As noise increases, the market moves from left to right, thereby crossing the **TPx1** phase boundary; this results in falling liquidity as buy orders prevail and price goes up. The basin of attraction is the Triple Point. Note that in the certainty region information traders prevail.

The upper left corner is a region of overpricing, news about fundamentals may suggest higher volatility of expected cash flows. Rational investors with adequate valuation will detect overpricing and increasingly place sell orders, thereby bringing the market back down to the Triple Point, that is, into the "sell" region; liquidity falls, and price adjusts to the equilibrium market-risk level **TPy1**.

Over- or underpricing is a short-lived phenomenon after news (new relevant information) is released. In both cases, moving from the "wait" phase either through sell or buy to reach the attractor point (Triple Point = dynamic equilibrium) sees the market cross the phase boundaries. If there is nonlinearity present, there should be a function that experiences abrupt change with a small change in the X-axis (noise/information trading). This also should be related to a jump in the price, as the market quickly adjusts to new information and incorporates the news in the price. (Documented overreactions to news can be modeled with a longer route to the **Triple Point** after crossing the phase boundaries.)

In our model, at the phase boundary, liquidity[2] experiences an abrupt change with only a small change in the ratio of noise to rational trading (Figure 4). Lillo and Farmer [2005] empirically demonstrate that liquidity, not large volume, determines large price changes; therefore, changed liquidity at the phase boundary will result in the jumps that are often observed in stock prices. As we move horizontally on the phase diagram (Figure 3) from "wait" to "buy" in the region of underpricing, liquidity increases as more noise traders are present. When we cross into the "buy" region characterized by positive demand, the liquidity function changes. (Please, note movements in the "wait" region could be only horizontal, by definition.)

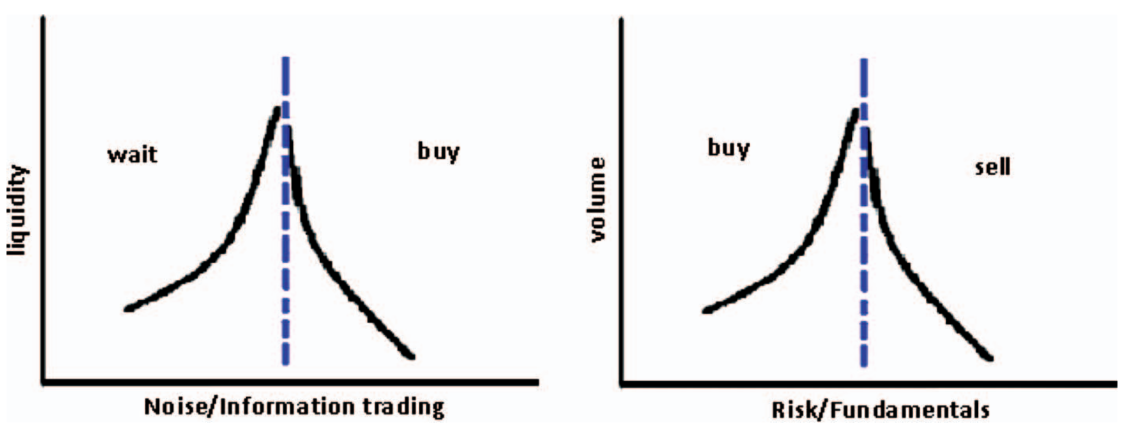

FIGURE 4 Liquidity increases until the phase boundary and experience discontinuity at phase transition. (Color figure available online).

Similarly, in the overpriced region, when a rational investor wants to sell at *x* price, the market should move horizontally to find a "noise" buyer. To the right the ratio of noise traders' increases, increasing the ease of trade (liquidity). After the phase boundary, however, the market is characterized by negative demand and a sell order faces lower liquidity, that is, fewer buyers.

According to EMH, these are short-lived anomalies that are arbitraged away. The simultaneous execution of a large number of trades produces efficient outcomes, and a dynamic equilibrium among the three states is present. Also, according to EMH, all investors are rational and base their decisions on fundamental values. Trading only occurs due to liquidity-need investors, who take the opposite side of the trade. As noted earlier, we define those seeking liquidity as "noise" traders since they are not trading based on information.

From Triple Point to Critical Point

Having shown the Triple Point to be an effective attractor basin, we still need to find a mechanism that will explain the empirical record of extreme events evidenced by stock markets, one that far exceeds Gaussian distributed returns (e.g., Mandelbrot and Hudson [2004], Baum and McKelvey [2006]). Sornette [2003a] presents a general theory of financial crashes and stock market instabilities, asserting that markets exhibit complex organization and dynamics. Moreover, he suggests that large scale patterns of a catastrophic nature result from global cooperative process over the whole system by repetitive interaction. A power-law distribution punctuated with log-periodic oscillations in the index prices seems to be the signature of an impending crash. Among many other examples, Baum and McKelvey [2006] show evidence of power-law distribution in the daily log returns of Dow Jones and NASDAQ and argue that observed power laws stem from nonindependent behavior that is ever present in social contexts (including stock markets).

A normal market sustains dynamic equilibrium around the Triple Point where prices reflect all available information about fundamentals. The market incorporates new information into the stock prices efficiently, and security mispricing is temporary. Information does not create ambiguity, noise in the market is offset by rational decision making, and anomalies are short lived. When the level of noise increases, information ambiguity prevails, and the market moves to the right in the region of uncertainty; efficient information processing and probabilistic decision making becomes more difficult. According to Knight [1921], "risk" refers to a situation in which the probability of an outcome can be determined, and therefore the outcome insured against. "Uncertainty," by contrast, refers to an event whose probability cannot be known. Knight's distinction between risk and uncertainty differentiates between the measurability or objectivity of probability.

The region of uncertainty, characterized by ambiguity of information, spans from a critical level of noise (**R1**) to the right. Rational probabilistic decision making is impeded, and the bimodal demand function (Plerou et al. [2003]) signals herding behavior. At **R1**, information complexity impacts risk-taking behavior, and so the market moves to a higher risk/fundamentals region. Information cascading, herding, rule-based trading, etc., create a complex network (self-organization) among traders and leads to a power-law distribution of returns. Therefore, we define this region as "fractal." Figure 3 now consists of four regions: Certainty, Uncertainty, Risk, and Fractal. We briefly define each of these regions as follows:

A. *Certainty*. Most pronounced at the *Origin*. Note the points on each axis where the risk/fundamentals and noise/information ratios equal 1, that is, **TPy1** and **TPx1**. On the X-axis, accurate information dominates noise. On the Y-axis, risk—as measured by the volatility of firm's underlying fundamentals—is low relative to that used in the valuation process. Rational traders unanimously agree on the under pricing the closer to the origin the market moves.
B. *Uncertainty*. To the right of the Triple Point location on the X-axis, **TPx1**, traders lose contact with any reliable means of attaching true value to information about a particular stock/company. Complexity and ambiguity of information take over the market. While uncertainty keeps increasing towards **R2**, the market becomes vulnerable to chaos, that is, bifurcations due mainly to external anomalies hitting a market. Price continuation anomalies are increasing as uncertainty levels increase.
C. *Risk*. Above the location of the Triple Point on the Y-axis, **TPy1**, traders move away from simply trading based on knowledge about the *current* "fundamental" value of a stock or firm to start betting on *future* value. A rational bubble emerges when market price depends on its own expected rate of change reminiscent of the models of Blanchard [1979] and Blanchard and Watson [1982]. Risk increases up to the location of the Critical Point on the y-axis. Above this point we show "Chaotic Risk"; this is the point where risk taking becomes vulnerable to chaos—bifurcations that can set off significant crashes.

Note that we show Knight's [1921] risk, uncertainty, and certainty as juxtaposed at the Triple Point. This is the core explanation underlying EMH—traders leaning toward all three situations trade concurrently with quick adjustments of the market shifting toward one or the other of the three conditions.

D. *Fractal*. The region between the Triple and Critical Points is notable for increasingly dramatic volatility incidents. Since there is growing evidence that many of these incidents follow fractal patterns, we label the region Fractal. This region corresponds to the bubble regime (Sornette [2003a]) where nonstationary increasing volatility

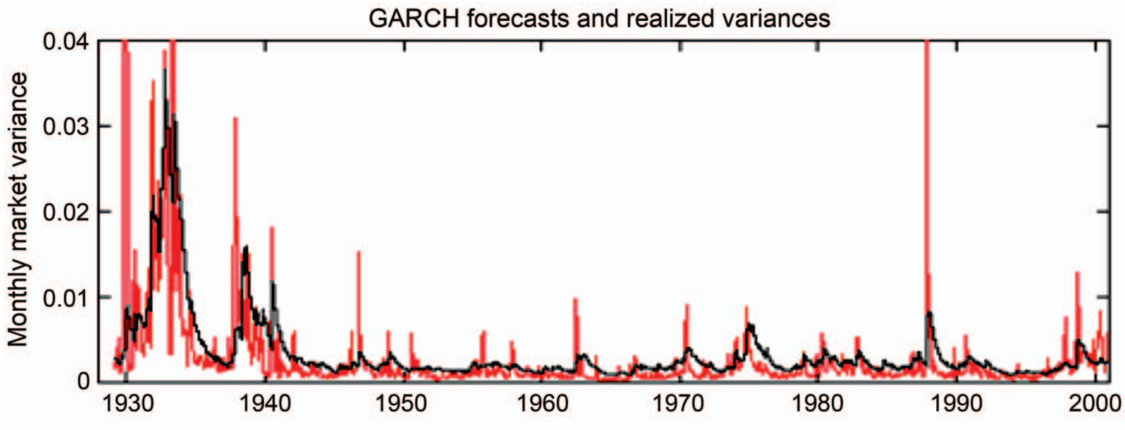

FIGURE 5 Depiction of volatility incidents above the GARCH line. (Color figure available online).

correlations are reported. Moreover, the regime switches between "normal" and "bubble," comprising a dynamical model that recovers all the stylized facts of empirical prices. This is what we focus on next.

How much time does the Dow Jones, for example, spend in the fractal region? Note from Figure 3 that, as risk and uncertainty increase, traders end up in the fractal region. "Tradition" in finance holds that it spends most of the time at the Triple Point. For empiricists this is represented by generalized autoregressive conditional heteroskedasticity (GARCH) (Bollerslev [1986]). But as one can see in Figure 5, there are many market variances well above the "GARCH line" (which is in black in the figure). Mandelbrot (in Mandelbrot and Hudson (2004, p. 13]) calculates that

> . . .by the conventional wisdom, August 1998 simply should never have happened. . .. The standard theories. . .would estimate the odds of that final, August 31, collapse, at one in 20 million—an event that, if you traded daily for nearly 100,000 years, you would not expect to see even once. The odds of getting three such declines in the same month were even more minute: about one in 500 billion (p. 4). . .. [An] index swing of more than 7 percent should come once every 300,000 years; in fact, the twentieth century saw forty-eight such days.

Empirical evidence of market efficiency depicts the dynamic equilibrium around the Triple Point while power laws in return distribution and volatility autocorrelation functions reveal the incidences of different market dynamics that are better examined at various scales. Events in this "Fractal" region should not be simply labeled as "random anomalies." It seems obvious that the argument between Fama et al. and Mandelbrot et al. is passé. It is time to pay equal attention to both regions. As market moves into the fractal region, synchronization of trading increases and log-periodic oscillations leading to the Critical point are observed. The costs stemming from what happens in the fractal region, however, are less frequent, but not as infrequent as some would want us to believe, and the costs of ignoring this region are very high.

Volatility

In the EM paradigm volatility compares to molecular Brownian motion (dead things with no memory), while long range dependence (power laws in autocorrelation functions) has been detected in financial time series.

In the EM paradigm, where volatility compares to molecular Brownian motion (dead things with no memory), we now have considerable research indicating both a transition point and then subsequent bubble buildup—that is, volatility dynamics—appearing as power laws in autocorrelation functions and/or log-periodicity (e.g., Zhou and Sornette (2002, 2003], Sornette and Zhou [2006], Masakawa [2007], Du and Ning [2008], Eom et al. [2008], Kumar and Deo [2009], Yan, Woodard, and Sornette [2010]). Research on the *scaling behavior of volatility,* as indicated by volatility power law distributions, explains price changes at different time horizons—hourly, daily, weekly, and monthly—and reveals vertical dependence that is explained by the existence of traders with different time horizons.

Gençay et al. [2009] show that in heterogeneous markets, low-frequency shocks penetrate though all layers to the short-term traders, while high frequency shocks appear to be short lived. This explains the patterns of volatility observed in endogenous shocks as a result of self-organization (underlying chaotic dynamics, i.e., the period doubling, bifurcation state). Sornette and Helmstetter's [2003] article pointing to the cumulative effect of small shocks fits here as well. Coarse-grained volatility at low frequency captures the views and actions of long-term traders while fine-grained

TABLE 1
Scale Free Theories Explaining Nonlinear Trader Behaviors.

Self-organized criticality	*Sandpiles; forests; heartbeats*: Under constant tension of some kind (gravity, ecological balance, delivery of oxygen), some systems reach a critical state where they maintain adaptive stasis by preservative behaviors—such as sand avalanches, forest fires, changing heartbeat rate, species adaptation—which vary in size of effect according to a power law (Bak [1996]).
Interacting fractals	*Food web; firm & industry size*: The fractal structure of a species is based on the food web (S. Pimm quoted in Lewin [1992], p. 121), which is a function of the fractal structure of predators and niche resources (Preston [1948], Pimm [1982], Solé et al. [2001], West [2006]).

volatility at high frequency captures the views and actions of short-term traders.

The following scale-free theories explain volatility best. We give the basic definitions in Table 1 and then apply them to the context of firms underlying stock-trading behaviors.

Self-organized criticality

Volatility is simply prices changes that range from many small movements to a few large movements, with a crash being the largest. This process is exactly what Bak [1996] emphasizes in his "self-organized criticality" theory—the many small to few large change movements that keep the slope of a sand pile at a certain angle, are seen in all sorts of processes whereby a particular functionally adaptive position is maintained—sand piles, species, markets, and so on. Podobnik et al. [2006] find that well-working "transition" economies in Eastern Europe, for example, show a power-law distribution of stock price movements. This is what we should see as stock markets in transition adjust to underlying fundamentals and exogenous changes impacting firms and trader behaviors.

Interacting fractals

Since we know that U.S. manufacturing firms, for example, are power-law distributed, and that many industries are as well (e.g., Stanley et al. [1996], Takayasu and Okuyama [1998], Axtell [2001], Ishikawa [2006], Andriani and McKelvey [2009], Chou and Keane [2009], Cirillo and Husler [2009], Glaser [2009], Zhang, Chen, and Wang [2009]), the valuation basis of trading behavior occurs in the context of interacting fractal structures, such that the "predator-prey" relationships (e.g., McKelvey et al. [2010] list 19 predator-prey studies in biology) (seen as mergers and acquisitions and other competitive activities in the business world; Park et al. [2009]) underlie the multifractal volatility incidents we see in market behavior.

Rule Collapse; LeBaron's Model

LeBaron [2001a, b] shows that as investment rules co-evolve toward a single super-rule, the market collapses. This is a second-order phase transition characterized in the model by vanishing liquidity and a flight to supposed quality—traders gravitate toward the same buy-rule, thinking it is best. Second-order phase transitions show a discontinuity in the second derivative of free energy. As we described earlier, in systems containing liquid and gaseous phases, there exists a special combination of pressure and temperature, known as the Critical Point, at which the transition between liquid and gas becomes a second-order transition. Near the Critical Point, the fluid is sufficiently hot and compressed so that the distinction between the liquid and gaseous phases is almost nonexistent and rapid oscillations can occur from one to the other. In our model, at the Critical Point, all traders have the same opinion—"sell"—as suggested by Johansen and Sornette [1998]. The collapse of the market under these conditions is consistent with its characterization as a dissipative structure that releases energy to achieve the "more ordered" state of uniform opinion and EMH behavior.

We suggest an explanation of the differences in relaxation. If we let the system reach ***R*2** (in Figure 3) and "self-crash," the system enters the chaos state and the preceding small shocks continue to influence the dynamics of the volatility. Relaxation to the unconditional average-volatility state is anomalously slow as compared to external shocks (a) that can be economic (Russians default on their bonds) or (b) shocks created by a piece of really bad news (airplanes fly into the World Trade Center; short term but not long-term economic impact). In this fractal regime one can attempt to stop the spiral up toward ***R*2** by inducing a shock *before* the system reaches ***R*2**—this gets the system back to dynamic equilibrium more quickly and with less negative economic consequences.

If a system enters the chaotic regime at ***R*2**, its unstable behavior is transient (though in the Great Depression and in the current liquidity induced meltdown, the crash lasts several years), and eventually the system returns to stable economic and EMH conditions. However, as current arguments in financial-economic discourse indicate, there is little consensus about what to do to more quickly dissipate the behavior of chaotic systems, so we cannot (here) give solid details about effective intervention strategies, though we realize the Obama Administration is surely trying.

MODEL IMPLICATIONS FOR RESEARCH METHODOLOGY

We propose a phase-transition model of market dynamics that allows for extreme events and explains the origin of power laws in the returns distribution as well as PLs of the autocorrelation function. In essence this model accounts for conditions of increased information complexity (and/or noise) when EMH-based decision making is impeded and traders resort to imitation and herding.

Below the critical value of information complexity. ***R*1**, the net demand for stocks is roughly zero; neither buying nor selling predominates, which agrees with the dynamic stability in the basin of attraction—that is, at our Triple Point of normal efficient market behavior, EMH, at equilibrium. Above the Triple Point (i.e., above **TPx1**) we show the Critical Noise tipping point, ***R*1** (in Figure 3), where the bimodal distribution of buy and sell limit orders emerges with the two most probable values symmetrical around zero demand as reported by Plerou et al. [2003).

Above this ***R*1** threshold of noise, self-organization in the market leads to formation of clusters of traders, whose activity synchronizes and the emergence of a dominant trading rule leads to a crash, as shown by LeBaron [2001a]. Our model also explains the precrash log-periodic patterns of index prices documented by different groups of physicists (cf., Yalamova [2003, 2010]).

Moreover, our model calls for different research methods in the two regimes. Below ***R*1**, EMH assumes random changes in the stock prices and normal distributions of returns, as well as fast decay of the autocorrelation function. In the regime above the ***R*1** threshold, emerging behavior results in PLs in returns and long memory (i.e., slow decay) in the autocorrelation function. PLs describe empirical scaling relationships that are emergent quantitative patterns of (a) structure or (b) dynamics that are self-similar or fractal-like over many orders of magnitude. Since the Hurst exponent is related to the fractal dimension, which gives a measure of the roughness of a surface (e.g., Struzik [2001], Yalamova [2003, 2010], Grech and Mazur [2004], Cajueiro and Tabak [2004], Masakawa [2007], Alvarez-Ramirez et al. [2008], Eom et al. [2008]), it has emerged as a reliable indicator of the ***R*1** tipping point between EMH tracing and the PL-defined bubble buildup leading to the Critical Point and a market crash.

The relationship between the fractal dimension, D, and the Hurst exponent, H, is

$$D = 2 - H$$

The Hurst exponent provides a measure of whether the data are a pure random walk ($H = \frac{1}{2}$) or have underlying trends. When an autocorrelation has a very long (or mathematically infinite) decay, the series indicates a presence of a *long memory.* We come back to the Hurst exponent later on.

Power Laws of Returns

When EMH applies, prices follow random walks; prices are unpredictable as their changes efficiently incorporate randomly arriving information to the market. The logarithm of prices measures returns and follows a Gaussian distribution. There is sufficient empirical evidence showing that extreme events lying outside the three standard deviation limit allowed by the normal distribution are observed much too frequently to be deleted as random anomalies (Mandelbrot and Hudson [2004]); they also note that in stock trading, daily returns that are seven standard deviations away from the mean occur 10 million times more often than would be the case for random data.

The most popular way of showing PLs in the distribution of returns is to plot the log of the number of observations against the log of the return size and thereby produce in inverse sloping line that is the PL indication. This method has been questioned (Clauset et al. [2007]) and has received some critical remarks for being overused, but even so, the finance literature provides ample evidence, with precise methodology, documenting the existence of PLs in the distribution of index-price returns. In a subsequent paper, Yalamova [2010] describes and compares the accuracy of different methodologies for time-scale invariance with special emphasis on wavelets. Interested readers are also referred to Yalamova [2003] for a preview.

Power Laws of The AutoCorrelation Function (ACF)

In the EMH regime, returns are unpredictable because they do not show discernable correlation. Time-series analysis of these prices detects very fast (exponential) decay of the ACF Volatility measured by the variance of the stock returns has short memory, that is, essentially random. Time series are modeled as random and independent using the AutoRegressive Moving Average process (ARMA). Moreover, extended time series analysis defines the AutoRegressive Integrated Moving Average (ARIMA) model as a generalization of an ARMA model. Within the realm of EMH, integer integration order faces no objection.

Allowing for fractional-integration leads into the world of fractals that creates the division between efficient market and the econophysics paradigm (e.g., West and Deering [1995], Mantegna and Stanley [2002], Vasconcelos [2004], Rosser [2008]). In a fractal market regime, returns are still not predictable, but volatility *is* predictable (Poon and Granger [2003]). Classic econometrics (e.g., ARCH, GARCH) shows the clustering of volatility but does not capture long-memory in the volatility of returns unless it allows for fractional integration in the models (e.g., FIGARCH).

Fractality, a sign of structural changes taking place in a market, will show up in the long memory of the volatility. The hyperbolic (slow) decay of the ACF reveals dependence, correlation, and feedback among traders, which can lead to imitation, herding, and rule-based trading. Our hypothesis is that information becomes expensive and/or complex and ambiguous, such that the market cannot find its way back to equilibrium.

Grossman and Stiglitz [1980] argue that if information gathering is costly, a competitive Walrasian market does not always remain in equilibrium. Moreover, they show that when the EMH is true and information is costly, competitive markets break down. Informed traders realize they can stop paying for information and still do as well as uninformed traders. Therefore, having some fraction of informed traders does not necessarily produce equilibrium. Having no one informed is not equilibrium either, if each trader thinks that there are profits to be made from becoming informed.

Following Grossman and Stiglitz, we do not reject EMH but rather wish to extend it to fit conditions when information is complex, that is, costly and/or problematic with respect to market analysis. As information complexity increases, analysis becomes more costly, and imitation in trading (herding) is more desirable, if not optimal; following an emerging rule that others are following is cheaper than trying to get access to new information. The long memory in the volatility of stock returns has already found its way into econometrics models such as ARFIMA and FIGARCH. Given that many time series exhibit slowly decaying autocorrelations, the advantages of Autoregressive Fractionally Integrated Moving Average models with hyperbolic autocorrelation decay seems clear in our proposed framework.

Though it seems quite logical and expedient to embrace the entire spectrum of integrated processes, one can only wonder why there is so much resistance to accepting fractals as part of the financial-asset pricing models. Among the reasons might be the dense covariance matrix they create or that the process does not possess the martingale property. Our view is that these are minor and avoidable technical details. More importantly, there is as yet no unifying theory that allows for both efficient market equilibrium and anomalies that break through the threshold of information complexity, which then create conditions for self-organization and interdependence among the market participants, that is, correlated behaviors. Information cascades, herding, and feedback loops are not part of an EMH-style market presumably composed of independent rational investors.

A logarithmic-scale plot of the autocorrelation function gives the same visual illustration of PLs as described in our explanation of PL distributions above. The autocorrelation function ρ takes a power-law form with constant C and exponent α:

$$\rho(s) = Cs^{-\alpha}$$

The fractional integration parameter α of the ACF is related to the volatility scaling (Hurst) exponent as:

$$H = 1 - \alpha/2$$

Volatility is random if the Hurst exponent is equal to $\frac{1}{2}$, which indicates totally random, so-called "Brownian motion" movements. Volatility persistence as measured by the Hurst exponent increases above $\frac{1}{2}$ during periods of increased information complexity. More specifically, if $H > \frac{1}{2}$, rule-based trading, herding, imitation, and increased mutual influence among traders leads to log-periodic oscillations of prices appearing as precursory bubble build-up patterns before crashes, that is, during the buildup between Triple and Critical Points in Figure 3.

CONCLUSION

When transparent information about fundamental firm values exists, rational investors find undervalued stocks to buy; for the most part they then exercise a buy-and-hold strategy. This trading approach appears in the region near the "Certainty" region near the Origin in our Figure 3. There is little trading in this zone because values are known; there is a lack of supply at these prices and not much trading because traders will sell only in case of liquidity needs. Buying pressure moves the market to dynamic equilibrium around the Triple Point in Figure 3. Here, trading conforms to the efficient market hypothesis. Prices quickly adjust to fair value; the coexistence of some certainty over prices, along with noise and risk (the three phases in trading) keeps a normally functioning market in the EMH basin of attraction. We see this as the juxtaposition and rapid oscillation among Knight's [1921] elements of risk, uncertainty, and certainty. At this point, fair value prevails, noise trading equals information trading, and the average net demand is 0. At this level, we have very simple rules, information is freely available and investors are rational with unlimited abilities to process information. The market quickly adjusts to new information, anomalies are short lived, and noise levels are modest. Under these conditions, linear models can provide appropriate approximations. EMH also states that it is not possible to beat the market, since arbitrage opportunities are short lived and any anomalies are randomly distributed. Net demand is zero and the distribution of orders is symmetric around zero

In reality, of course, institutions and individual investors, in attempts to beat the market, introduce hedging strategies, trading rules, derivative securities, etc., and the complexity of the financial trading system increases. The foregoing strategies rely on trading approaches based on "rules" or formulas that appear to reduce noise and allow greater risk. As trading complexity rises, then, heterogeneous agents switch increasingly from information-based trading to "rule-governed behavior" such that the ratio of *noise traders to information traders* and pursuit of *risk rather than fundamental values* increases. We draw on a literature showing that increased noise in the market leads to bimodal net demand distribution (Plerou et al. [2003]) and the buildup of bubbles. Bubbles occur when noise- and risk-based trading increase as a result of an initiating event (usually new technology like the laptop computer or new formulas like derivatives or options pricing models) and the market moves away from the EMH basin of attraction. However, the market most often reverts back to the Triple Point (a soft landing) many times before conditions materialize that allow some trading rule to begin to dominate such that the market moves into the fractal region in Figure 3 and on not so rare occasions (Mandelbrot and Hudson [2004]) keeps the bubble buildup going up to the Critical Point when the bubble bursts.

We note that LeBaron's [2001a, b] computational model shows that loss of heterogeneity results in market crashes; one also sees this in the Long-Term Capital Management (LTCM) failure as traders more and more learned what LTCM was doing (Lowenstein [2000]). With the increase of noise trading, prices destabilize and periodic orbits emerge as demand distribution bifurcates as imitation and information cascade amplify. The boundary between buy and sell states is crossed multiple times forming log-periodic oscillations in the price. Coevolution of trading rules towards one super rule (LeBaron [2001b]) leads to order in the market as all traders adopt the same decision and the market collapses (Johansen and Sornette [1998]). Although the empirical evidence of patterns before drawdown is increasing, there has been no attempt to build a theory that reconciles the EMH with these extreme "anomalies" in a more systematic and structured framework. Physicists (e.g., Sornette et al. [1996], Johansen et al. [2000]) show various power-law distributed returns and log-periodic oscillations of prices occurring between the Triple and Critical Points. Note, however, that the numerical methods used by these researchers have been criticized for "fitting several free parameters in noisy data" (e.g., Sornette [2003a, b]), that is, considered inaccurate or data mining as well as lacking theoretical background.

Once traders lose their heterogeneity by learning from each other and from market results what the best rules and formulas appear to be, what began as a "tiny initiating event" (Holland [1995, 2002]) in the form of an experimental new investment strategy spirals up into a widespread belief about how best to win out over noise and take increasingly leveraged but seemingly well-defined risks. Convergence of thousands of traders on a particular formula—what is really convergence toward a single buy-sell rule—for some period of time sets the bubble-creation process in motion until some intervening event disrupts it and the market may not reach the Critical point. Growing clusters of imitation and herding among market participants create a regime of trading synchronization exhibiting log-periodic oscillations in index levels as shown in Sornette (2003b). At the Critical Point "sell" orders prevail precipitating a market crash.

We suggest a couple of scale-free theories, drawn from Andriani and McKelvey [2009], to explain the various trader-behaviors serving to move a market away from equilibrium (the Triple Point), when a specific level of information ambiguity $\boldsymbol{R}\mathbf{1}$(noise) in Figure 3 is reached. The result is that the self-organization of traders toward a formula-based buy-sell rule (usually with the help of computer-based modern quantitative methods) leads to loss of heterogeneity. Since all of the scale-free theories explain Pareto-distributed observations, our contribution is to offer a scale-free theory explanation for the log-periodic patterns that Sornette [2003a, b, 2004], Sornette et al. [1996, 2003), Zhou and Sornette [2002, 2003], and Yan et al. [2010] have detected with various numerical tests for both bubble increase and decreases between Triple and Critical Points as well as for volatilities after a crash at the Critical Point.

Moreover, we can move the model to a larger scale of the financial system and following several crashes in different sectors explain the clustering and synchronization leading to a major system wide crisis. In a chapter in a forthcoming book (McKelvey and Yalamova [forthcoming]), we illustrate in some detail the initial underpinnings of the 2007 liquidity crisis and the rise and fall of the bubble based on mortgage-based investment securitization packages and overleveraged derivative formulas. We also go back to the invention of derivatives circa 1973 and trace developments forward to 2007. We show how various kinds of phenomena well explained by various scale-free theories spiraled up from a "*tiny initiating event*" (Holland [1995, 2002]) to the so-called "*Minsky Moment*"—a market crash point—and then crashed back down and into the worldwide recession we see in 2009 going into 2010. The great irony in all the above is that the sequence of trader behaviors leading up to the 2007 Minsky Moment—that is, *everyone using derivatives, everyone using high leverage, the China effect on bond prices, the repeal of the Glass-Steagall Act, everyone using mortgage-based securities, everyone using subprime fixed-interest five-year "teaser" loans*—should have been so obvious (e.g., Cooper [2008], Morris [2008], Phillips [2008], Baker [2009], Cohan [2009], Foster and Magdoff [2009]) that even the dumbest Wall Street and Federal Reserve types or other financial experts should have seen all this unfolding throughout the eight years of the G. W. Bush Administration without the added insights brought on by knowledge of the scale-free theories we mention—*this particular Minsky Moment should have never occurred*.

We believe it is time for financial experts to pay equal attention *both* (a) to theories endemic to EMH (Fama [1970]), the dominant paradigm in finance, and (b) the fractal dynamics of Mandelbrot [1963, 1997] and Sornette [2003b] among other econophysicists pertaining to the fractal region developing between our Triple and Critical Points of Figure 3. In retrospect, we see, from our story line about what led up to the 2007 liquidity crisis (McKelvey and Yalamova [forthcoming]), for example, that so much must have been known by the financial experts on Wall Street and even in the U.S. government that the progression toward the crash should have been foreseen and negated.

We attempt to bring together the growing literature pertaining to the fractal region. But we also bring in a more explicit application of complexity science and a number of scale-free theories stemming from econophysics (e.g., West and Deering [1995], Mantegna and Stanley [2000], Vasconcelos [2004], Newman [2005], Andriani and McKelvey [2009]) to better explain the financial dynamics appearing in works by Sornette [2003b]. See also Yalamova [2003] for a review of nonlinear patterns before crashes, as researched by different groups of physicists (e.g., Vandewalle et al. [1998], Gluzman and Yukalov [1997, 1998]). EMH is not a "shaky myth" as Mandelbrot is want to say (Mandelbrot and Hudson [2004, p. ix]). But on the other hand, fractal finance is not a shaky myth perpetrated by false profits either. Fractal dynamics occur far more often than EMH folks want to accept, costs individuals and countries far more than it should, and well-known diagnostics and solutions are left on the back burner—ignored when they should not be. What we bring to light in this paper is the ***R*1** tipping point between EMH trading behavior and the beginning of bubble buildups by traders' interactive learning, herding, and rule-based trading, which gives rise to power-law described volatility distributions.

NOTES

1. For example, in November, in the Normandy region of France, as the sun rises in the morning, one can see snow, rain, clouds, and sunshine all within half an hour, with no wind movement at all. Not quite simultaneous, but almost! But if the sun doesn't come out, there can be oscillation between snow, rain, and vapor (i.e., just clouds).
2. Liquidity in finance, not to be confused with the liquid state in physics, is determined by the amount of orders on the opposite side of the trade. It means how "easily" one can find a buyer or a seller at the price one is willing to trade at.

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